

Functions and Their Graphs

3.4 Graphing Techniques: Transformations

1. B 2. E 3. H 4. D

5. I 6. A 7. L 8. C

9. F 10. J 11. G 12. K

13. $y = (x - 4)^3$ 14. $y = (x + 4)^3$ 15. $y = x^3 + 4$ 16. $y = x^3 - 4$

17. $y = -x^3$ 18. $y = -x^3$ 19. $y = 4x^3$ 20. $y = \frac{1}{4}x^3 = \frac{x^3}{64}$

21. (1) $y = \sqrt{x} + 2$
 (2) $y = -(\sqrt{x} + 2)$
 (3) $y = -(\sqrt{-x} + 2)$

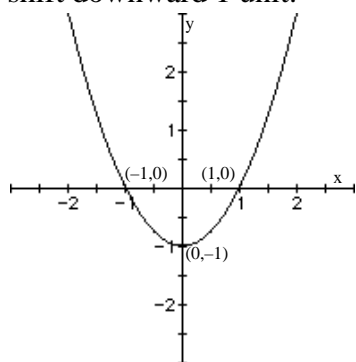
22. (1) $y = -\sqrt{x}$
 (2) $y = -\sqrt{x - 3}$
 (3) $y = -\sqrt{x - 3} - 2$

23. (1) $y = -\sqrt{x}$
 (2) $y = -\sqrt{x} + 2$
 (3) $y = -\sqrt{x + 3} + 2$

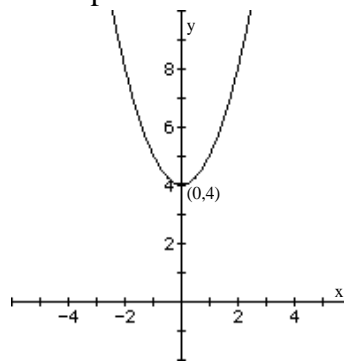
24. (1) $y = \sqrt{x} + 2$
 (2) $y = \sqrt{-x} + 2$
 (3) $y = \sqrt{-(x + 3)} + 2 = \sqrt{-x - 3} + 2$

25. C 26. D 27. C 28. A

29. $f(x) = x^2 - 1$
 Using the graph of $y = x^2$, vertically shift downward 1 unit.

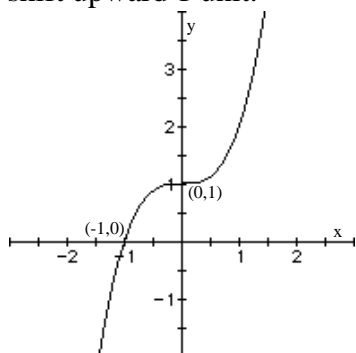


30. $f(x) = x^2 + 4$
 Using the graph of $y = x^2$, vertically shift upward 4 units.



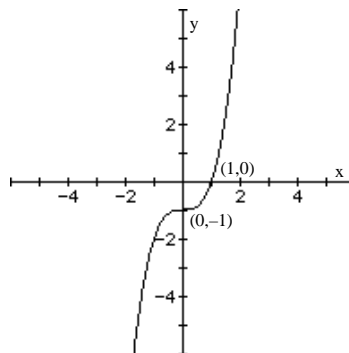
31. $g(x) = x^3 + 1$

Using the graph of $y = x^3$, vertically shift upward 1 unit.



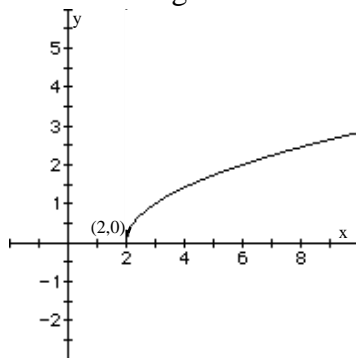
32. $g(x) = x^3 - 1$

Using the graph of $y = x^3$, vertically shift downward 1 unit.



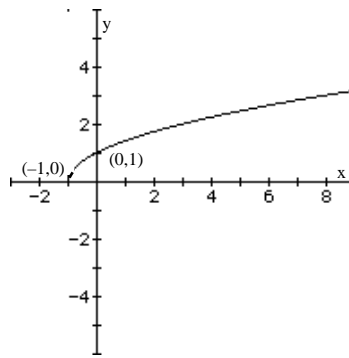
33. $h(x) = \sqrt{x-2}$

Using the graph of $y = \sqrt{x}$, horizontally shift to the right 2 units.



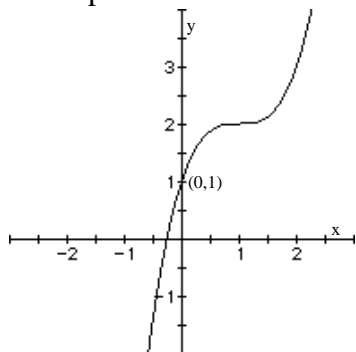
34. $h(x) = \sqrt{x+1}$

Using the graph of $y = \sqrt{x}$, horizontally shift to the left 1 unit.



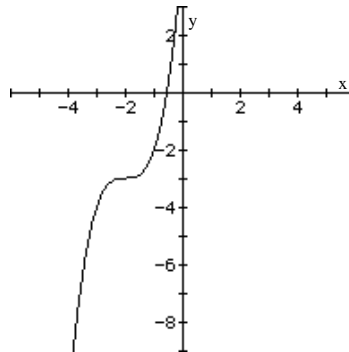
35. $f(x) = (x-1)^3 + 2$

Using the graph of $y = x^3$, horizontally shift to the right 1 unit, then vertically shift up 2 units.



36. $f(x) = (x+2)^3 - 3$

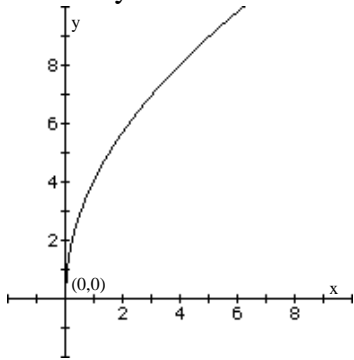
Using the graph of $y = x^3$, horizontally shift to the left 2 units, then vertically shift down 3 units.



Section 3.4 Graphing Techniques: Transformations

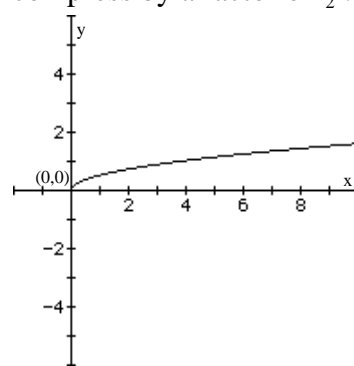
37. $g(x) = 4\sqrt{x}$

Using the graph of $y = \sqrt{x}$, vertically stretch by a factor of 4.



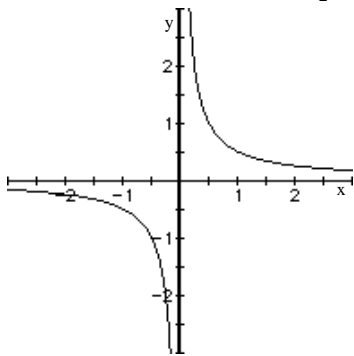
38. $g(x) = \frac{1}{2}\sqrt{x}$

Using the graph of $y = \sqrt{x}$, vertically compress by a factor of $\frac{1}{2}$.



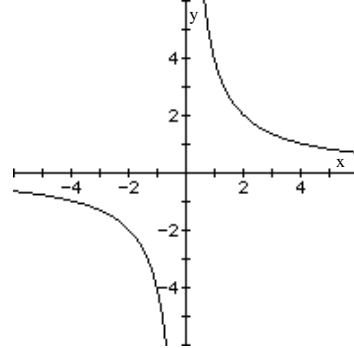
39. $h(x) = \frac{1}{2x}$

Using the graph of $y = \frac{1}{x}$, vertically compress by a factor of $\frac{1}{2}$.



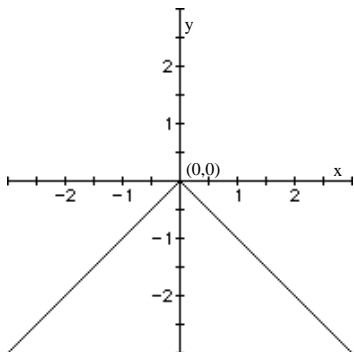
40. $h(x) = \frac{4}{x}$

Using the graph of $y = \frac{1}{x}$, vertically stretch by a factor of 4.



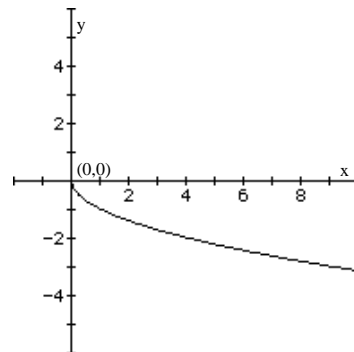
41. $f(x) = -|x|$

Reflect the graph of $y = |x|$, about the x-axis.



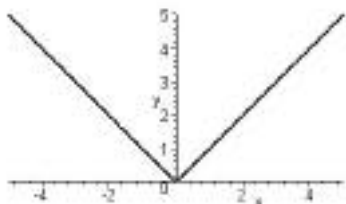
42. $f(x) = -\sqrt{x}$

Reflect the graph of $y = \sqrt{x}$, about the x-axis.



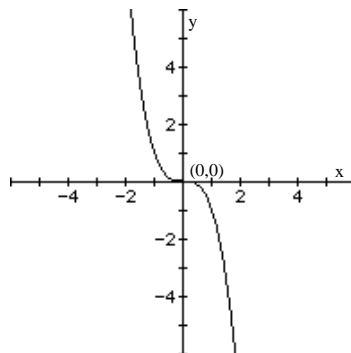
43. $g(x) = -|x|$

Reflect the graph of $y = |x|$, about the y-axis.



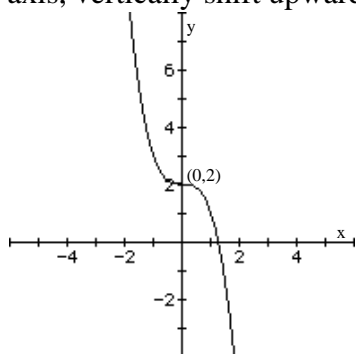
44. $g(x) = -x^3$

Reflect the graph of $y = x^3$, about the x-axis.



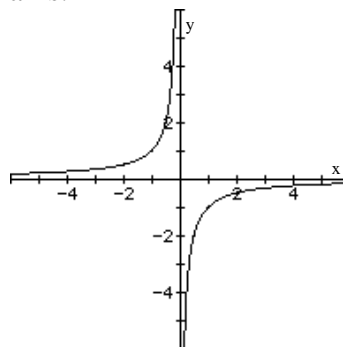
45. $h(x) = -x^3 + 2$

Reflect the graph of $y = x^3$ on the x-axis, vertically shift upward 2 units.



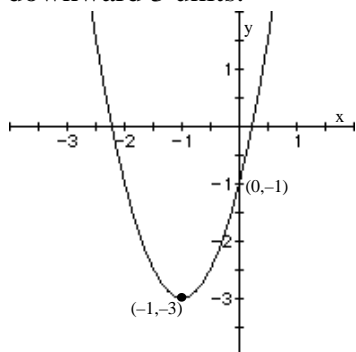
46. $h(x) = \frac{1}{-x}$

Reflect the graph of $y = \frac{1}{x}$, about the y-axis.



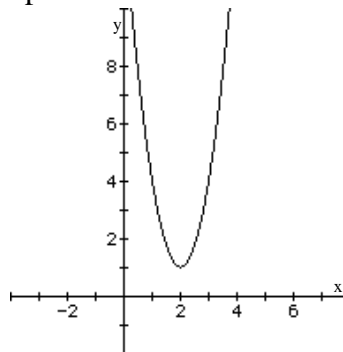
47. $f(x) = 2(x+1)^2 - 3$

Using the graph of $y = x^2$, horizontally shift to the left 1 unit, vertically stretch by a factor of 2, and vertically shift downward 3 units.



48. $f(x) = 3(x-2)^2 + 1$

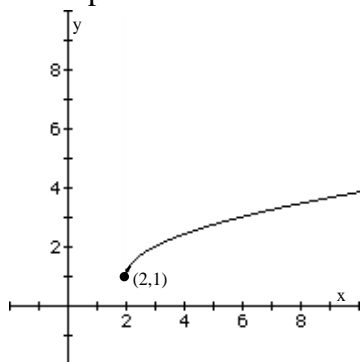
Using the graph of $y = x^2$, horizontally shift to the right 2 units, vertically stretch by a factor of 3, and vertically shift upward 1 units.



Section 3.4 Graphing Techniques: Transformations

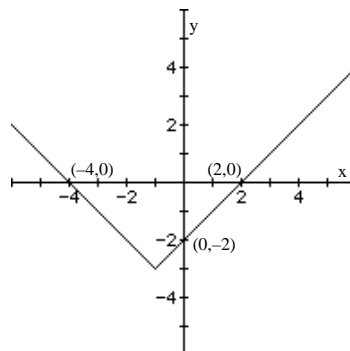
49. $g(x) = \sqrt{x-2} + 1$

Using the graph of $y = \sqrt{x}$, horizontally shift to the right 2 units and vertically shift upward 1 unit.



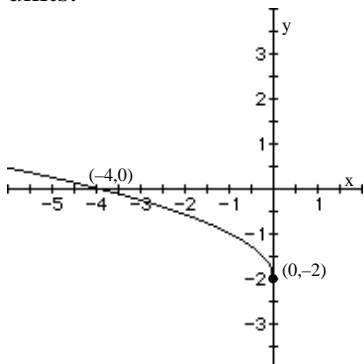
50. $g(x) = |x+1| - 3$

Using the graph of $y = |x|$, horizontally shift to the left 1 unit and vertically shift downward 3 units.



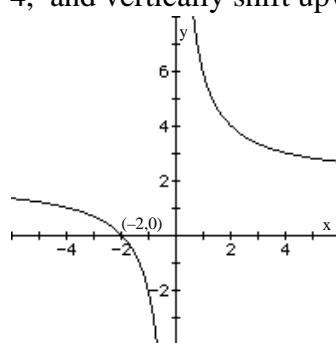
51. $h(x) = \sqrt{-x} - 2$

Reflect the graph of $y = \sqrt{x}$, about the y-axis and vertically shift downward 2 units.



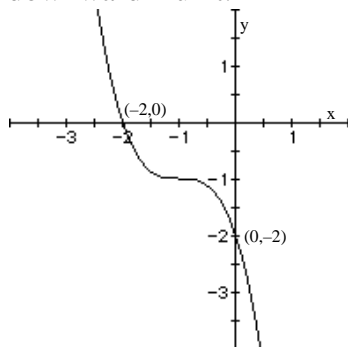
52. $h(x) = \frac{4}{x} + 2$

Stretch the graph of $y = \frac{1}{x}$ by a factor of 4, and vertically shift upward 2 units.



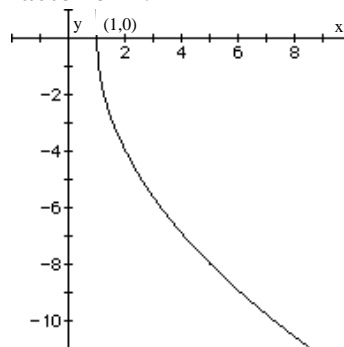
53. $f(x) = -(x+1)^3 - 1$

Using the graph of $y = x^3$, horizontally shift to the left 1 units, reflect the graph on the x-axis, and vertically shift downward 1 unit.



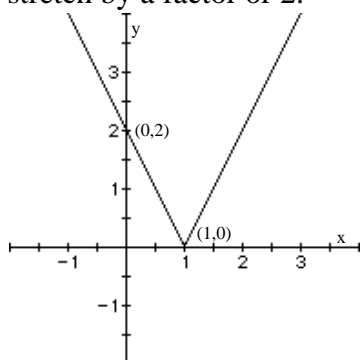
54. $f(x) = -4\sqrt{x-1}$

Using the graph of $y = \sqrt{x}$, horizontally shift to the right 1 unit, reflect the graph on the x-axis, and stretch vertically by a factor of 4.



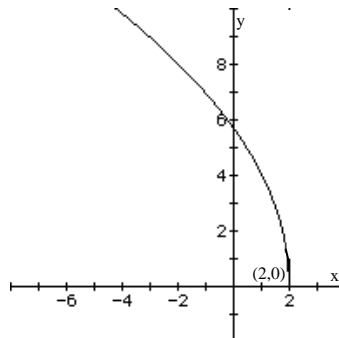
55. $g(x) = 2|1 - x| = 2|x - 1|$

Using the graph of $y = |x|$, horizontally shift to the right 1 unit, and vertically stretch by a factor of 2.



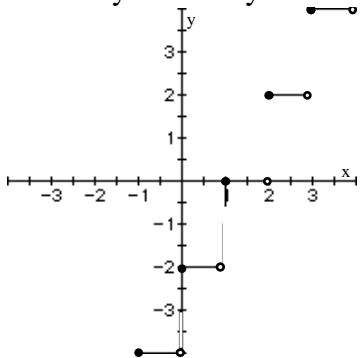
56. $g(x) = 4\sqrt{2 - x} = 4\sqrt{-(x - 2)}$

Reflect the graph of $y = \sqrt{x}$ on the y -axis, horizontally shift to the right 2 units, and vertically stretch by a factor of 4.



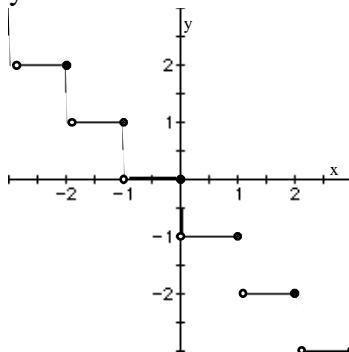
57. $h(x) = 2\text{int}(x - 1)$

Using the graph of $y = \text{int}(x)$, horizontally shift to the right 1 unit, and vertically stretch by a factor of 2.

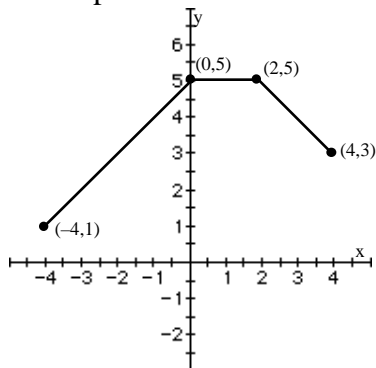


58. $h(x) = \text{int}(-x)$

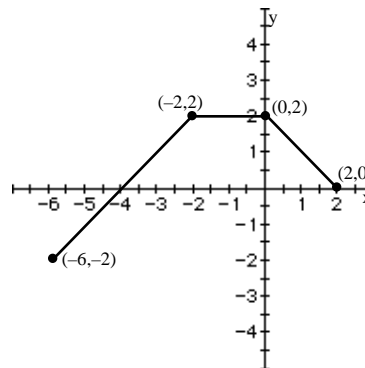
Reflect the graph of $y = \text{int}(x)$, about the y -axis.



59. (a) $F(x) = f(x) + 3$
Shift up 3 units.



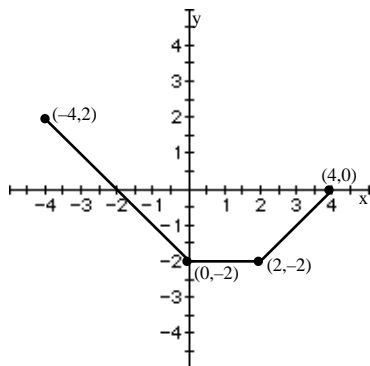
(b) $G(x) = f(x + 2)$
Shift left 2 units.



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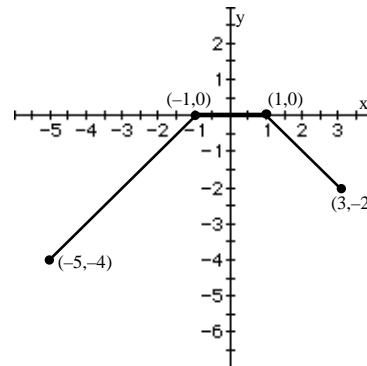
(c) $P(x) = -f(x)$

Reflect about the x-axis.



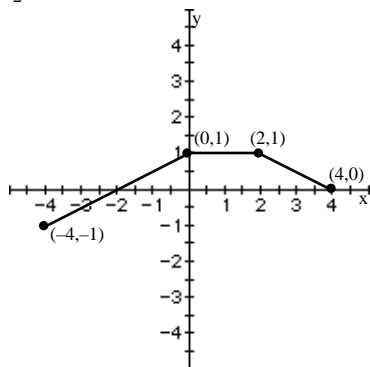
(d) $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



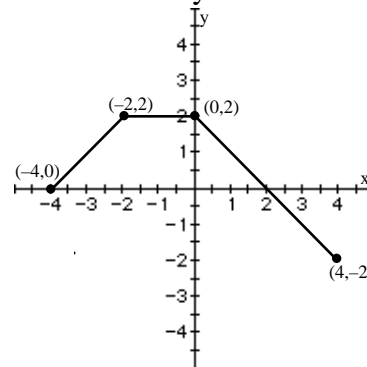
(e) $Q(x) = \frac{1}{2}f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



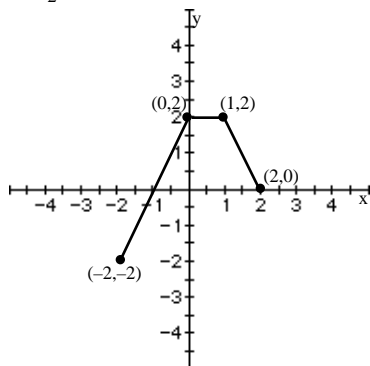
(f) $g(x) = f(-x)$

Reflect about y-axis.



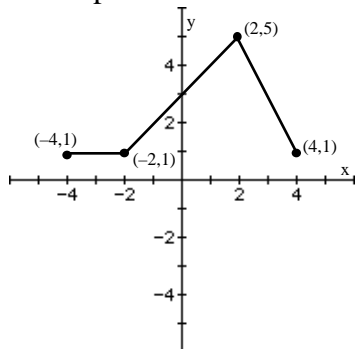
(g) $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



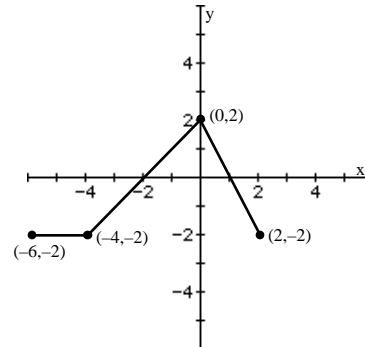
60. (a) $F(x) = f(x) + 3$

Shift up 3 units.



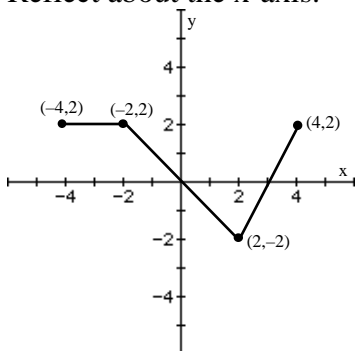
(b) $G(x) = f(x + 2)$

Shift left 2 units.



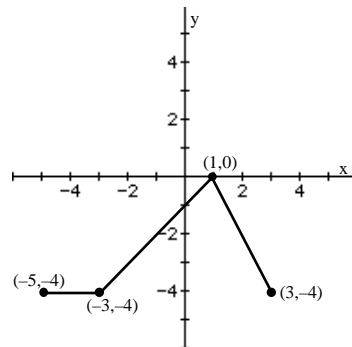
(c) $P(x) = -f(x)$

Reflect about the x-axis.

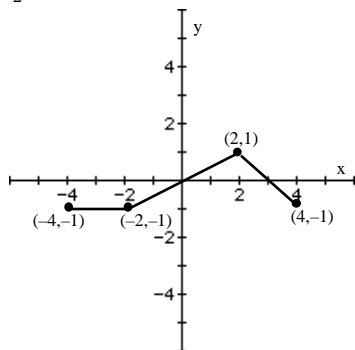


(d) $H(x) = f(x + 1) - 2$

Shift left 1 unit and shift down 2 units.

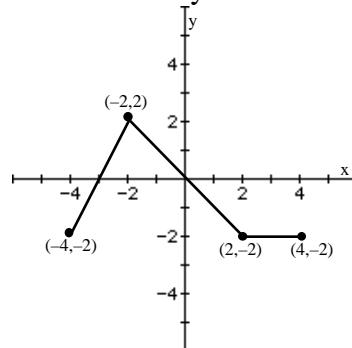


(e) $Q(x) = \frac{1}{2}f(x)$

Compress vertically by a factor of $\frac{1}{2}$.

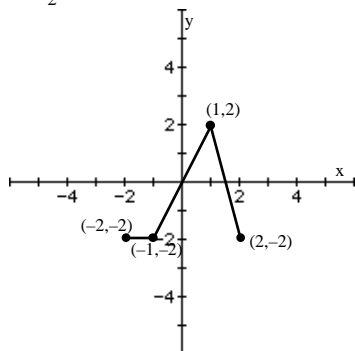
(f) $g(x) = f(-x)$

Reflect about y-axis.

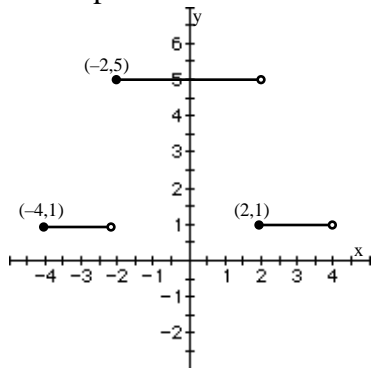


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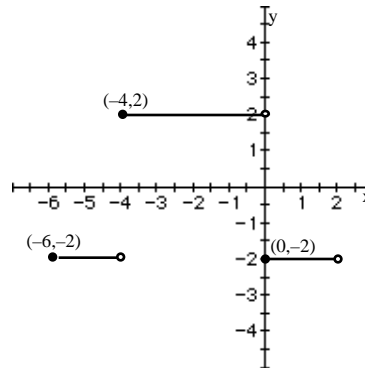
- (g) $h(x) = f(2x)$
Compress horizontally by a factor of $\frac{1}{2}$.



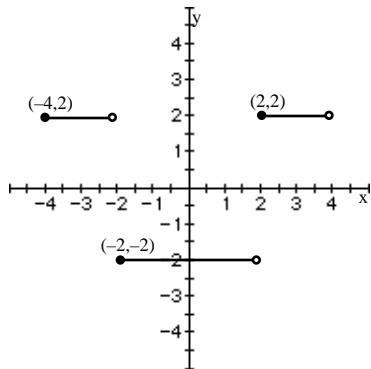
61. (a) $F(x) = f(x) + 3$
Shift up 3 units.



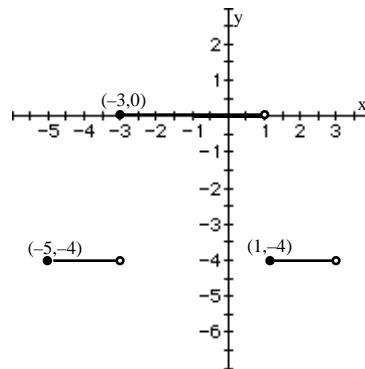
- (b) $G(x) = f(x + 2)$
Shift left 2 units.



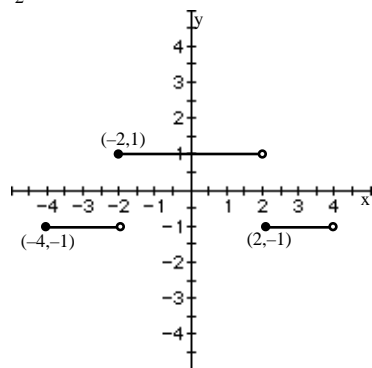
- (c) $P(x) = -f(x)$
Reflect about the x-axis.



- (d) $H(x) = f(x + 1) - 2$
Shift left 1 unit and shift down 2 units.

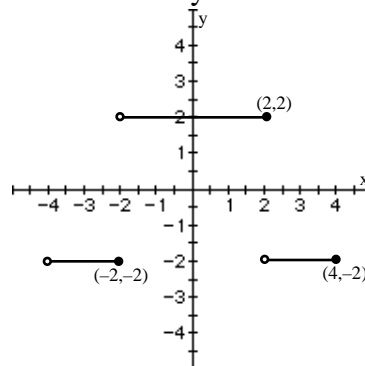


(e) $Q(x) = \frac{1}{2}f(x)$

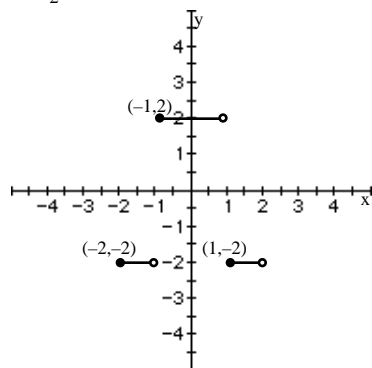
Compress vertically by a factor of $\frac{1}{2}$.

(f) $g(x) = f(-x)$

Reflect about y-axis.

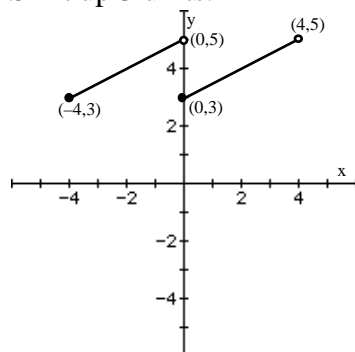


(g) $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.

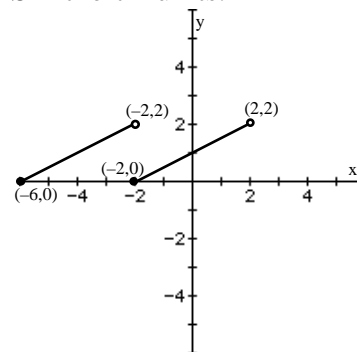
62. (a) $F(x) = f(x) + 3$

Shift up 3 units.



(b) $G(x) = f(x + 2)$

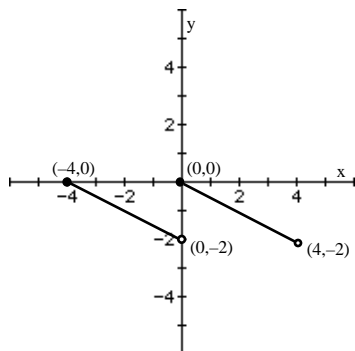
Shift left 2 units.



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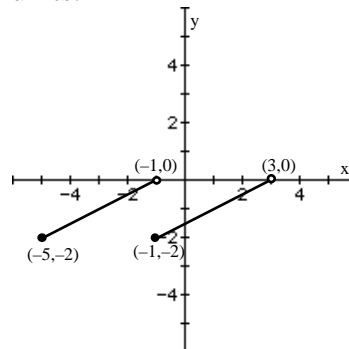
(c) $P(x) = -f(x)$

Reflect about the x-axis.



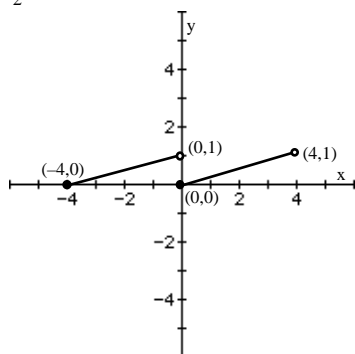
(d) $H(x) = f(x + 1) - 2$

Shift left 1 unit and shift down 2 units.



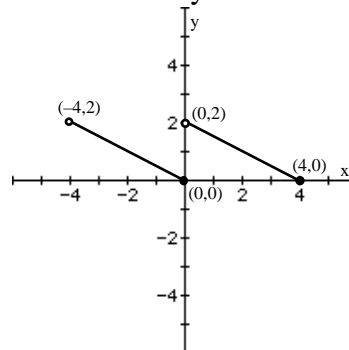
(e) $Q(x) = \frac{1}{2}f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



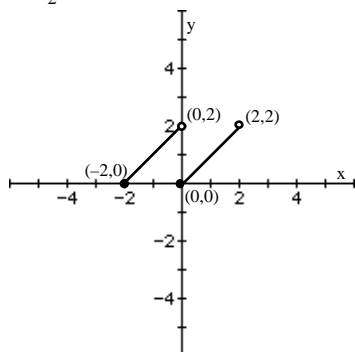
(f) $g(x) = f(-x)$

Reflect about y-axis.

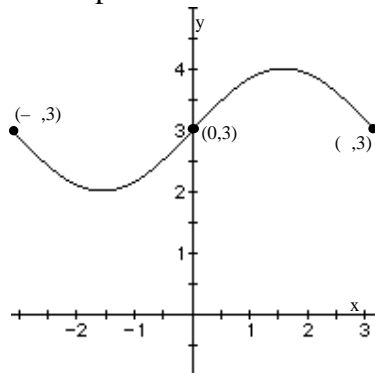


(g) $h(x) = f(2x)$

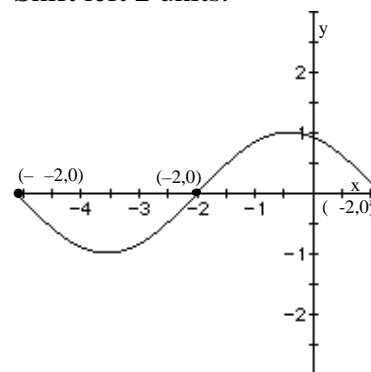
Compress horizontally by a factor of $\frac{1}{2}$.



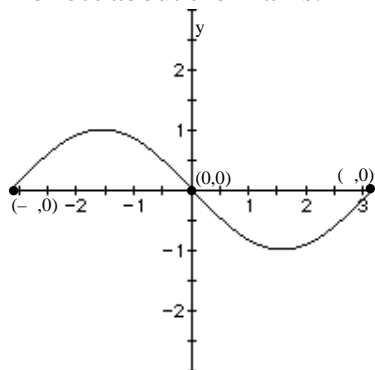
63. (a) $F(x) = f(x) + 3$
Shift up 3 units.



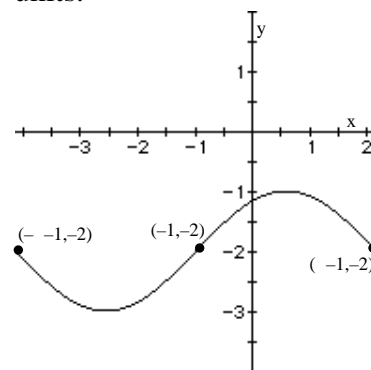
- (b) $G(x) = f(x + 2)$
Shift left 2 units.



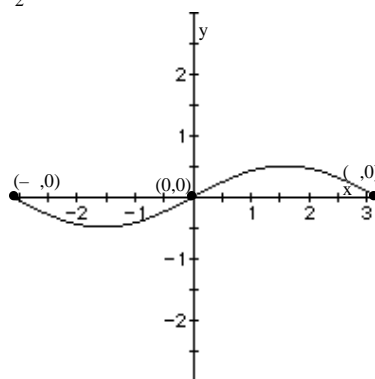
- (c) $P(x) = -f(x)$
Reflect about the x-axis.



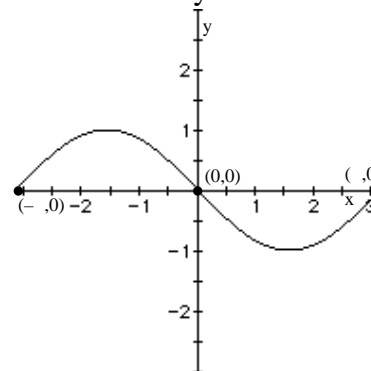
- (d) $H(x) = f(x + 1) - 2$
Shift left 1 unit and shift down 2 units.



- (e) $Q(x) = \frac{1}{2}f(x)$
Compress vertically by a factor of $\frac{1}{2}$.

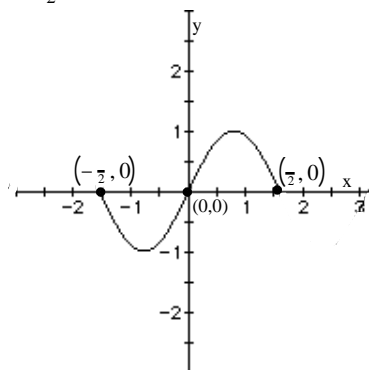


- (f) $g(x) = f(-x)$
Reflect about y-axis.

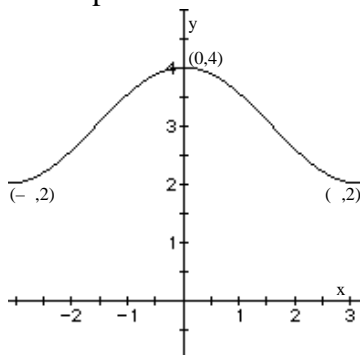


Section 3.4 Graphing Techniques: Transformations

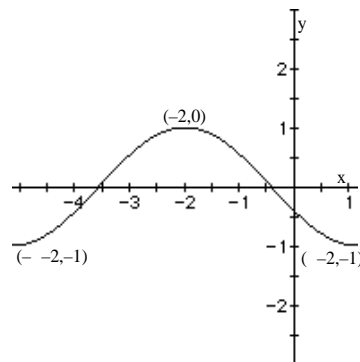
- (g) $h(x) = f(2x)$
Compress horizontally by a factor of $\frac{1}{2}$.



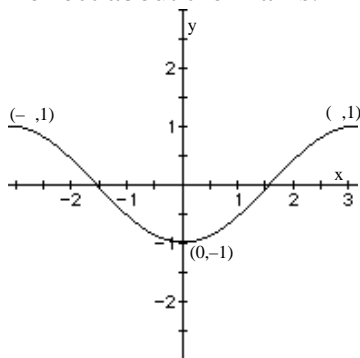
64. (a) $F(x) = f(x) + 3$
Shift up 3 units.



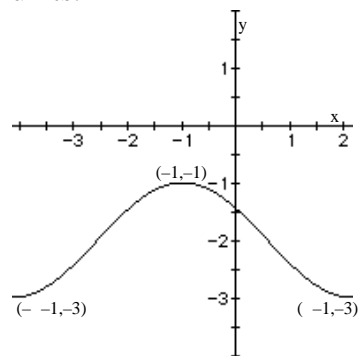
- (b) $G(x) = f(x + 2)$
Shift left 2 units.



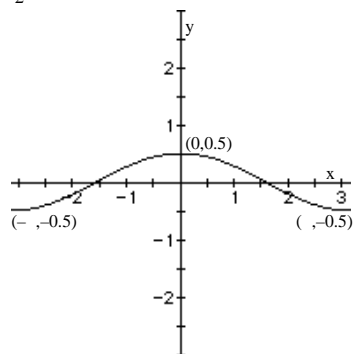
- (c) $P(x) = -f(x)$
Reflect about the x-axis.



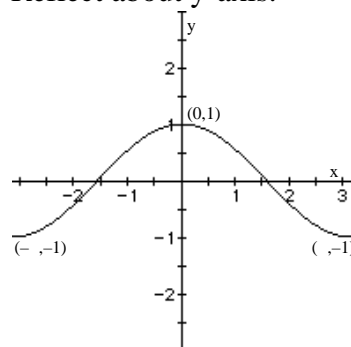
- (d) $H(x) = f(x + 1) - 2$
Shift left 1 unit and shift down 2 units.



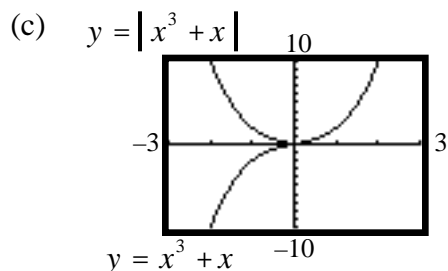
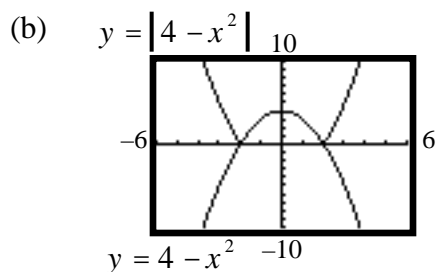
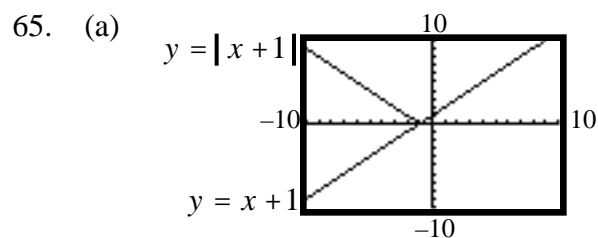
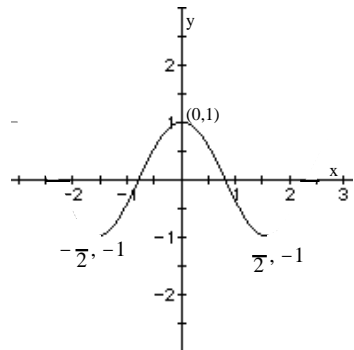
- (e) $Q(x) = \frac{1}{2}f(x)$
Compress vertically by a factor of $\frac{1}{2}$.



- (f) $g(x) = f(-x)$
Reflect about y-axis.

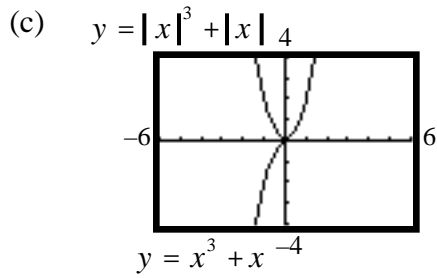
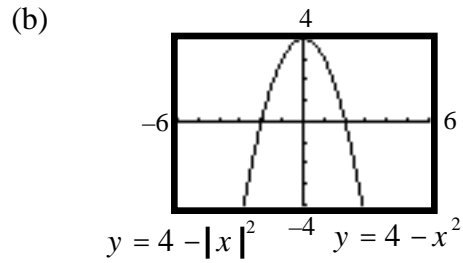
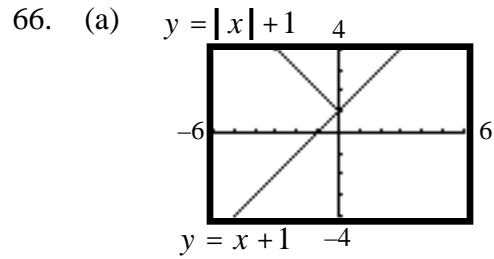


- (g) $h(x) = f(2x)$
Compress horizontally by a factor of $\frac{1}{2}$.

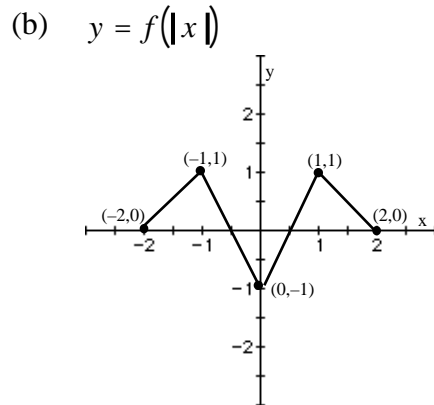
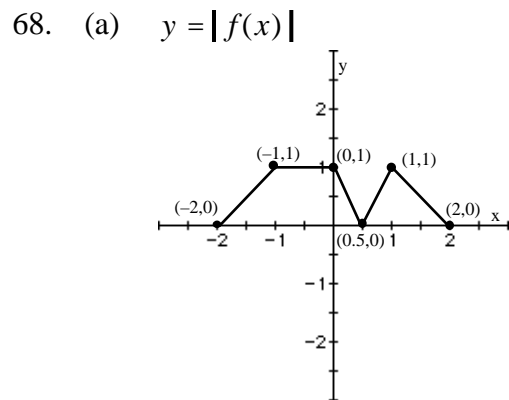
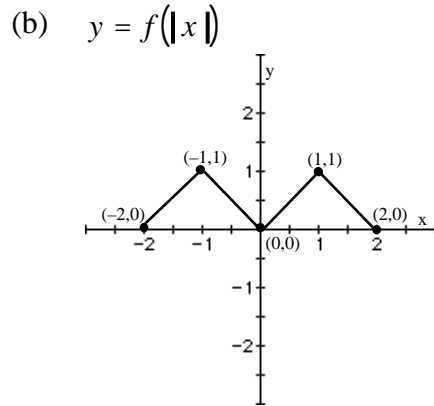
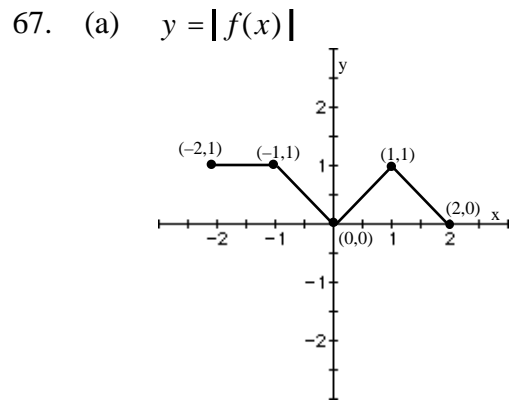


- (d) Any part of the graph of $y = f(x)$ that lies below the x-axis is reflected about the x-axis to obtain the graph of $y = |f(x)|$.

Section 3.4 Graphing Techniques: Transformations

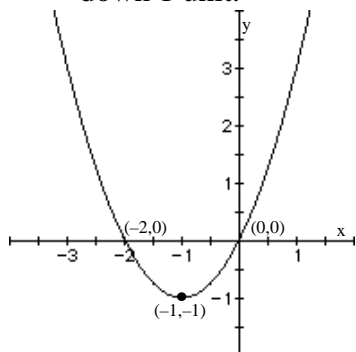


(d) Any part of the graph of $y = f(x)$ that lies to the right of the y -axis is reflected about the y -axis to obtain the graph of $y = f(|x|)$.



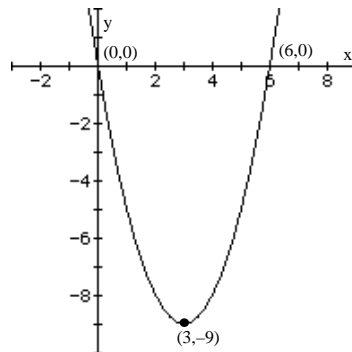
69. $f(x) = x^2 + 2x$
 $f(x) = (x^2 + 2x + 1) - 1$
 $f(x) = (x + 1)^2 - 1$

Using $f(x) = x^2$, shift left 1 unit and shift down 1 unit.



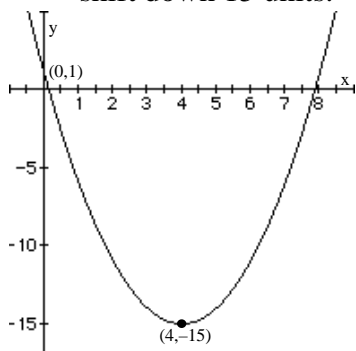
70. $f(x) = x^2 - 6x$
 $f(x) = (x^2 - 6x + 9) - 9$
 $f(x) = (x - 3)^2 - 9$

Using $f(x) = x^2$, shift right 3 units and shift down 9 units.



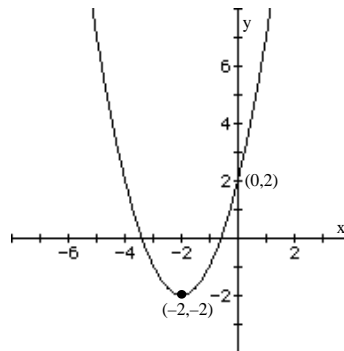
71. $f(x) = x^2 - 8x + 1$
 $f(x) = (x^2 - 8x + 16) + 1 - 16$
 $f(x) = (x - 4)^2 - 15$

Using $f(x) = x^2$, shift right 4 units and shift down 15 units.



72. $f(x) = x^2 + 4x + 2$
 $f(x) = (x^2 + 4x + 4) + 2 - 4$
 $f(x) = (x + 2)^2 - 2$

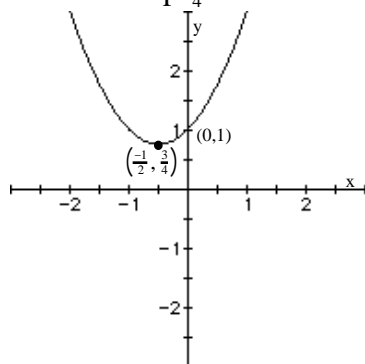
Using $f(x) = x^2$, shift left 2 units and shift down 2 units.



Section 3.4 Graphing Techniques: Transformations

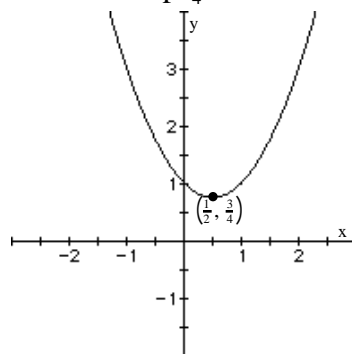
73. $f(x) = x^2 + x + 1$
 $f(x) = \left(x^2 + x + \frac{1}{4}\right) + 1 - \frac{1}{4}$
 $f(x) = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

Using $f(x) = x^2$, shift left $\frac{1}{2}$ unit and shift up $\frac{3}{4}$ unit.

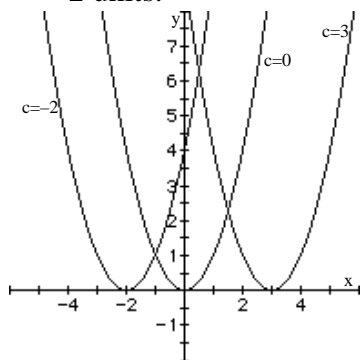


74. $f(x) = x^2 - x + 1$
 $f(x) = \left(x^2 - x + \frac{1}{4}\right) + 1 - \frac{1}{4}$
 $f(x) = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$

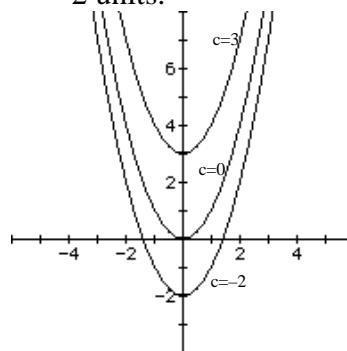
Using $f(x) = x^2$, shift right $\frac{1}{2}$ unit and shift up $\frac{3}{4}$ unit.



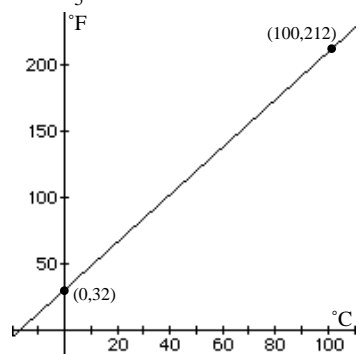
75. $y = (x - c)^2$
 If $c = 0$, $y = x^2$.
 If $c = 3$, $y = (x - 3)^2$; shift right 3 units.
 If $c = -2$, $y = (x + 2)^2$; shift left 2 units.



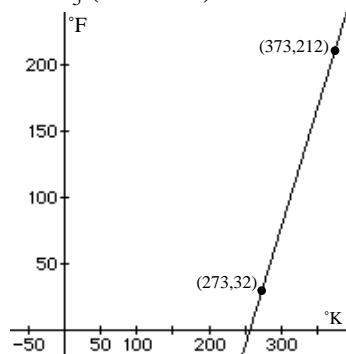
76. $y = x^2 + c$
 If $c = 0$, $y = x^2$.
 If $c = 3$, $y = x^2 + 3$; shift up 3 units.
 If $c = -2$, $y = x^2 - 2$; shift down 2 units.



77. $F = \frac{9}{5}C + 32$

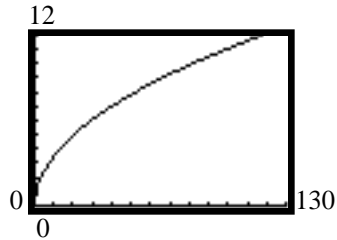


$F = \frac{9}{5}(K - 273) + 32$

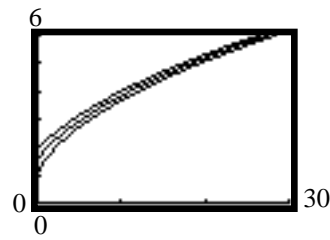


Shift the graph 273 units to the right.

78. (a) Graph:

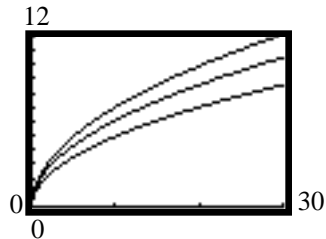


(b) Graph:

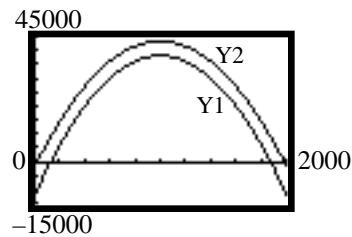


(c) As the length of the pendulum increases, the period increases.

(d) Graph:

(e) If the length of the pendulum is multiplied by k , the period is multiplied by \sqrt{k} .

79. (a)



(b) Select the 10% tax since the profits are higher.

(c) The graph of Y1 is obtained by shifting the graph of $p(x)$ vertically down 10,000. The graph of Y2 is obtained by multiplying the y-coordinate of the graph of $p(x)$ by 0.9. Thus, Y2 is the graph of $p(x)$ vertically compressed by a factor of 0.9.

(d) Select the 10% tax since the graph of $Y1 = 0.9p(x)$ $Y2 = -0.05x^2 + 100x - 6800$ for all x in the domain.